# A model for ultrarelativistic spherically symmetric Pre-Hawking radiating gravitational collapse

Miguel Piñol Ribas

March 3, 2013



**Abstract**

In this paper we present a simple but non-simplistic model of gravitational collapse with thermal emission of pre-Hawking radiation. We apply Einstein equations to a time-dependant spherically symmetric metric and an ultrarelativistic stress-energy tensor. In our model, particles either radially approach the center of the star as collapsing matter, or radially flee from it.

## 1 The Model

The more general metric for a spherically symmetric collapse presents the following structure:

$$d\tau^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 d\Omega^2, \tag{1}$$

where $\nu$ and $\lambda$ ara functions of radius $r$ and time $t$. It will be useful to define also a function $\phi$, so that

$$\nu = -\lambda - 2\phi \tag{2}$$

and

$$d\tau^2 = e^{-\lambda-2\phi} dt^2 - e^{\lambda} dr^2 - r^2 d\Omega^2, \tag{3}$$

as it may be demonstrated [Lan] that $\phi \geq 0$ all over to space (it is strictly equal to zero in the "outer space", outside the edge of the collapsing star, where the metric is identical to Schwarzschild's one). In addition, $\phi' \leq 0$ all over to space (we are denoting with $x'$ derivation in respect to $r$, as we will denote with $\dot{x}$ derivation in respect to $t$ all over the paper).

### 1.1 The Einstein equations

The Einstein equations for a metric of this kind are these ones:

$$8\pi T_1^1 = -e^{-\lambda}\left(\frac{\nu'}{r}+\frac{1}{r^2}\right)+\frac{1}{r^2}, \tag{4}$$

$$8\pi T_2^2 = 8\pi T_3^3 = -\frac{1}{2}e^{-\lambda}\left(\nu''+\frac{\nu'^2}{2}+\frac{\nu'-\lambda'}{r}-\frac{\nu'\lambda'}{2}\right)+\frac{1}{2}e^{-\nu}\left(\ddot{\lambda}+\frac{\dot{\lambda}^2}{2}-\frac{\dot{\lambda}\dot{\nu}}{2}\right) \tag{5}$$

$$8\pi T_0^0 = -e^{-\lambda}\left(\frac{1}{r^2}-\frac{\lambda'}{r}\right)+\frac{1}{r^2}, \tag{6}$$

$$8\pi T_0^1 = -e^{-\lambda}\frac{\dot{\lambda}}{r}. \tag{7}$$

The other components vanish identically.

## 1.2 The stress-momentum tensor

The stress-momentum tensor for a perfect fluid takes a simple form,

$$T_\beta^\alpha = g_{\beta\delta}\left(\rho+p\right)u^\alpha u^\delta - \eta_\beta^\alpha p, \tag{8}$$

where $\rho$ corresponds to the total energy density and $p$ to the pressure. For an ultrarelativistic fluid, $p$ is proportional to $\rho$:

$$p = \omega\rho \tag{9}$$

(concretely, it may be demonstrated that $\omega = \frac{1}{3}$, but we will keep on writing the constant of proportionality along the paper).

## 1.3 The four-velocity in our model

In our model, all particles (or "elements of mass", if you prefer) describe a purely radial trajectory (inwards, or outwards), so that the only non-null components of four-velocity are $u^0$ (which we are going to call $\gamma$, in order to obtain more beautiful expressions) and $u^1$.

From the metric, we can deduce the relationship between both components:

$$1 = e^{-\lambda-2\phi}\gamma^2 - e^{\lambda}(u_p^1)^2, \tag{10}$$

$$\left|u_p^1\right| = e^{-\lambda-\phi}\gamma\sqrt{1-e^{\lambda+2\phi}\gamma^{-2}} \tag{11}$$

(where the subindex $p$ stands for "particles").

For infalling particles, $u_p^r = -\left|u_p^1\right|$; for outgoing particles, $u_p^1 = +\left|u_p^1\right|$. Consequently, if we call $\chi$ the proportion of infalling particles, so that $1-d$ be the proportion of outgoing particles, the mean "r-velocity" $u^1$ will be the following one:

$$u^1 = \chi\left|u^1\right| + (1-\chi)(-\left|u^1\right|) = -(1-2\chi)e^{-\lambda-\phi}\gamma\sqrt{1-e^{\lambda+2\phi}\gamma^{-2}}. \tag{12}$$

### 1.4 The stress-energy tensor components

The stress-energy tensor components are, with all the considerations that we have taken until now, the following ones:

$$T_0^0 = e^{-\lambda-2\phi}\gamma^2 \left(\rho + p\right) - p = e^{-\lambda-2\phi}\gamma^2 \left(1 + \omega\right) \rho - \omega\rho, \tag{13}$$

$$T_1^1 = -e^{\lambda}\gamma u^1 \left(\rho + p\right) - p = (1 - 2\chi)e^{-\phi}\gamma^2 \sqrt{1 - e^{\lambda+2\phi}\gamma^{-2}} \left(1 + \omega\right) \rho - \omega\rho, \tag{14}$$

$$T_2^2 = T_3^3 = -p = -\omega\rho, \tag{15}$$

$$T_0^1 = e^{-\lambda-2\phi}\gamma u^1 \left(\rho + p\right) = -(1 - 2\chi)e^{-2\lambda-3\phi}\gamma^2 \sqrt{1 - e^{\lambda+2\phi}\gamma^{-2}} \left(1 + \omega\right) \rho. \tag{16}$$

### 1.5 The ultrarelativistic limit

In the ultrarelativistic limit, $\gamma >> 1$, so that we may ignore the terms with lesser powers of $\gamma$ in front of those in the higher ones:

$$T_0^0 \approx e^{-\lambda-2\phi}\gamma^2 \left(1 + \omega\right) \rho, \tag{17}$$

$$T_1^1 \approx (1 - 2\chi)e^{-\phi}\gamma^2 \left(1 + \omega\right) \rho, \tag{18}$$

$$T_0^1 \approx -(1 - 2\chi)e^{-2\lambda-3\phi}\gamma^2 \left(1 + \omega\right) \rho. \tag{19}$$

Therefore:

$$\rho \approx e^{\lambda+2\phi}\gamma^{-2} \left(1 + \omega\right)^{-1} T_0^0, \tag{20}$$

$$T_1^1 \approx (1 - 2\chi)e^{\lambda+\phi} T_0^0, \tag{21}$$

$$T_0^1 \approx -(1 - 2\chi)e^{-\lambda-\phi} T_0^0. \tag{22}$$

## 2 Solution of equations

### 2.1 Some considerations on the phases of collapse, in the light of pre-Hawking radiation

In 2006, Vachaspati, Stojkovic and Krauss demonstrated that collapsing stars emit pre-Hawking radiation [Vac]. While there exist some difference between both, pre-Hawking radiation spectrum results to be roughly proportional of Hawking radiation of black

holes. Thus, we are going to use a Hawking radiation-like expression for pre-Hawking radition in our model of gravitational col·lapse:

$$\dot{m}_{p-H} \approx \frac{-k}{r^2}, \tag{23}$$

where $k$ is a constant of proportionality, and $m$ denotes the inner "mass" which is lost as "radiation". In our model, we are not only going to consider the "global" emission of the collapsing star but also that of the "inner layers" towards the outer ones (and we will assume the same expression for that).

If we have into account the relationship between the function $\lambda$ and the "mass",

$$e^{\lambda} = 1 - \frac{2m}{r}, \tag{24}$$

we may straightforwardly deduce that

$$e^{-\lambda}\dot{\lambda} = \frac{2\dot{m}}{r}, \tag{25}$$

$$\dot{\lambda} = \frac{2\dot{m}}{r}e^{\lambda}. \tag{26}$$

On the other hand, from equations (7) and (22) we may obtain another expression for $\dot{\lambda}$:

$$\dot{\lambda} = -re^{\lambda}8\pi T_0^1 = r(1-2\chi)e^{-\phi}8\pi T_0^0, \tag{27}$$

which we may split into two terms in order to make explicit the contribution of infalling and outgoing fluxes:

$$\dot{\lambda} = r(1-\chi)e^{-\phi}8\pi T_0^0 - r\chi e^{-\phi}8\pi T_0^0. \tag{28}$$

Consequently, we may logically identify the second term in the previous equation with the pre-Hawking variation of $\lambda$ which we may obtain from equations (23) and (26):

$$\dot{\lambda} = \frac{2e^{\lambda}}{r}\left(\frac{-k}{r^2}\right), \tag{29}$$

$$-r\chi e^{-\phi}8\pi T_0^0 = \frac{2e^{\lambda}}{r}\left(\frac{-k}{r^2}\right), \tag{30}$$

$$\chi = \frac{e^{\lambda+\phi}}{4\pi r^2 T_0^0}\left(\frac{k}{r^2}\right). \tag{31}$$

From equations (27) and (31),

$$\dot{\lambda} = re^{-\phi}8\pi T_0^0 - \frac{2ke^{\lambda}}{r^3}. \tag{32}$$

In the light of equation (32), we may make some considerations on the phases of graviatational collapse:

1) In a first phase, $\lambda$ increases very fastly, due to the important flux of infalling matter and the insignificancy of pre-Hawking radiation.

2) When $\lambda$ reaches a certain value, we arrive to a phase of "stability", where the infalling flux of collapsing matter is exactly compensated by the outgoing flux of pre-Hawking radiation.

3) Finally, when the "outer layers" of infalling matter have already got exhausted, the pre-Hawking term prevails and $\lambda$ will diminish.

In this paper, we are going to focus our study mainly on the second phase.

## 2.2 The stability phase

In the stability phase, $e^{-\lambda} << 1$, so that

$$8\pi T_0^0 \approx \frac{1}{r^2} \tag{33}$$

(the approximations that we are going to perform in this section are not good for small values of $r$).

The condition for the stability phase consits on imposing $\dot{\lambda}_{st} = 0$:

$$0 = re^{-\phi_{st}}\left(\frac{1}{r^2}\right) - \frac{2ke^{\lambda_{st}}}{r^3}, \tag{34}$$

$$\phi_{st} = -\lambda_{st} + ln\left(r^2\right) - ln\left(2k\right). \tag{35}$$

In this situation,

$$\chi_{st} = \frac{1}{2}. \tag{36}$$

Consequently,

$$T_1^1 = 0, \tag{37}$$

$$0 = -e^{-\lambda_{st}}\left(\frac{\nu'_{st}}{r} + \frac{1}{r^2}\right) + \frac{1}{r^2}, \tag{38}$$

$$\frac{e^{\lambda_{st}}}{r^2} = \left(\frac{\nu'_{st}}{r} + \frac{1}{r^2}\right), \tag{39}$$

$$\nu'_{st} = \frac{e^{\lambda_{st}}}{r} - \frac{1}{r}, \tag{40}$$

$$\lambda'_{st} + 2\phi'_{st} = \frac{e^{\lambda_{st}} - 1}{r} \approx \frac{e^{\lambda_{st}}}{r}. \tag{41}$$

From equation (35),

$$\phi'_{st} = -\lambda'_{st} + \frac{2}{r} \approx -\lambda'_{st}. \tag{42}$$

From equations (41) and (42),

$$-\lambda'_{st} \approx \frac{e^{\lambda_{st}}}{r}, \tag{43}$$

$$\frac{d\lambda_{st}}{dr} \approx -\frac{e^{\lambda_{st}}}{r}, \tag{44}$$

$$e^{-\lambda_{st}} d\lambda_{st} \approx -\frac{dr}{r}. \tag{45}$$

By integrating,

$$-e^{-\lambda_{st}} \approx K_1 - ln(r), \tag{46}$$

$$\lambda_{st} \approx -ln\left(-K_1 + ln(r)\right). \tag{47}$$

### 2.3 On contour conditions

The function $\phi$ has a null value outside the edge of the collapsing star. If we call $M$ the total mass of the star, its radius $R$ is given by an expression of the following type [Pin]:

$$R = 2M + e^{\frac{f(r)-t}{2M}}, \tag{48}$$

where $M = M(t)$.
Thus,

$$\phi_{st}(R) = -\lambda_{st}(R) + ln\left(R^2\right) - ln\left(2k\right) = 0, \tag{49}$$

$$\lambda_{st}(R) = ln\left(R^2\right) - ln\left(2k\right) \approx -ln\left(-K_1 + ln(R)\right). \tag{50}$$

$$K_1 \approx lnR - \frac{2k}{R^2} \tag{51}$$

# 3 References

...

[Lan] Landau, L. D.; Lifshitz, E. M.; “The classical theory of Fields”. Course of Theoretical Physics Volume 2. University of Minnesota (1987)

[Vac] Vachaspati, T.; Stojkovic, D.; Krauss, L. M.; “Observation of incipient black holes and the information loss problem”. Phys. Rev. D 76:024005 (2007)

[Pin] Piñol, M.; Lopez, I.; ”Transition from Established Stationary Vision of Black Holes to Never-Stationary Gravitational Collapse”. http://arxiv.org/abs/1007.2734

...

contact: miguelpinol@comv.es